

Mathematics

Higher level

Paper 2

Friday 11 November 2016 (morning)

Candidate session number

2 hours

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.



4. [Maximum mark: 5]

Find the constant term in the expansion of $\left(4x^2 - \frac{3}{2x}\right)^{12}$.

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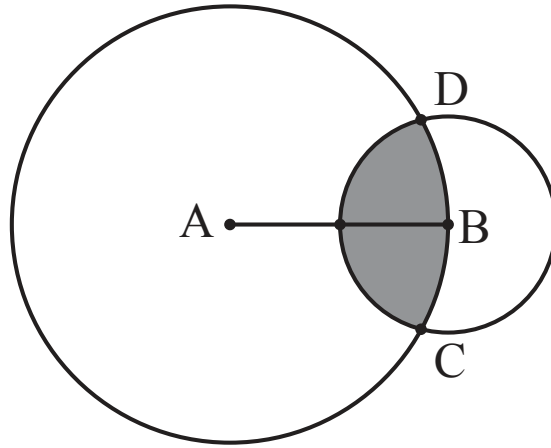


16EP05

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9. [Maximum mark: 8]

The diagram shows two circles with centres at the points A and B and radii $2r$ and r , respectively. The point B lies on the circle with centre A . The circles intersect at the points C and D .



Let α be the measure of the angle CAD and θ be the measure of the angle CBD in radians.

- (a) Find an expression for the shaded area in terms of α , θ and r . [3]
- (b) Show that $\alpha = 4 \arcsin \frac{1}{4}$. [2]
- (c) Hence find the value of r given that the shaded area is equal to 4. [3]

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16EP11

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 22]

Let the function f be defined by $f(x) = \frac{2 - e^x}{2e^x - 1}$, $x \in D$.

- (a) Determine D , the largest possible domain of f . [2]
- (b) Show that the graph of f has three asymptotes and state their equations. [5]
- (c) Show that $f'(x) = -\frac{3e^x}{(2e^x - 1)^2}$. [3]
- (d) Use your answers from parts (b) and (c) to justify that f has an inverse and state its domain. [4]
- (e) Find an expression for $f^{-1}(x)$. [4]
- (f) Consider the region R enclosed by the graph of $y = f(x)$ and the axes. Find the volume of the solid obtained when R is rotated through 2π about the y -axis. [4]



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11. [Maximum mark: 20]

A Chocolate Shop advertises free gifts to customers that collect three vouchers. The vouchers are placed at random into 10% of all chocolate bars sold at this shop. Kati buys some of these bars and she opens them one at a time to see if they contain a voucher. Let $P(X = n)$ be the probability that Kati obtains her third voucher on the n th bar opened.

(It is assumed that the probability that a chocolate bar contains a voucher stays at 10% throughout the question.)

(a) Show that $P(X = 3) = 0.001$ and $P(X = 4) = 0.0027$. [3]

It is given that $P(X = n) = \frac{n^2 + an + b}{2000} \times 0.9^{n-3}$ for $n \geq 3, n \in \mathbb{N}$.

(b) Find the values of the constants a and b . [5]

(c) Deduce that $\frac{P(X = n)}{P(X = n - 1)} = \frac{0.9(n - 1)}{n - 3}$ for $n > 3$. [4]

(d) (i) Hence show that X has two modes m_1 and m_2 .

(ii) State the values of m_1 and m_2 . [5]

Kati's mother goes to the shop and buys x chocolate bars. She takes the bars home for Kati to open.

(e) Determine the minimum value of x such that the probability Kati receives at least one free gift is greater than 0.5. [3]



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12. [Maximum mark: 18]

On the day of her birth, 1st January 1998, Mary’s grandparents invested $\$x$ in a savings account. They continued to deposit $\$x$ on the first day of each month thereafter.

The account paid a fixed rate of 0.4% interest per month. The interest was calculated on the last day of each month and added to the account.

Let $\$A_n$ be the amount in Mary’s account on the last day of the n th month, immediately after the interest had been added.

- (a) Find an expression for A_1 and show that $A_2 = 1.004^2x + 1.004x$. [2]
- (b) (i) Write down a similar expression for A_3 and A_4 .
- (ii) Hence show that the amount in Mary’s account the day before she turned 10 years old is given by $251(1.004^{120} - 1)x$. [6]
- (c) Write down an expression for A_n in terms of x on the day before Mary turned 18 years old showing clearly the value of n . [1]
- (d) Mary’s grandparents wished for the amount in her account to be at least \$20 000 the day before she was 18. Determine the minimum value of the monthly deposit $\$x$ required to achieve this. Give your answer correct to the nearest dollar. [4]
- (e) As soon as Mary was 18 she decided to invest \$15 000 of this money in an account of the same type earning 0.4% interest per month. She withdraws \$1000 every year on her birthday to buy herself a present. Determine how long it will take until there is no money in the account. [5]



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